

## **An Alternate Method to Estimate the *Pitzer* Coefficients of Apparent Molal Expansibility of Aqueous Electrolytes**

**Anil Kumar**

Department of Sugar Chemistry, Deccan Sugar Institute,  
Manjari (Bk) Pune 412 307, India

(Received 9 May 1987. Accepted 3 June 1987)

An alternate method to estimate the *Pitzer* coefficients of the ion-interaction model for apparent molal expansibility from the temperature dependence of the *Pitzer* coefficients for apparent molal volume is described. The calculated coefficients are in excellent agreement with those obtained using a least squares technique to experimental data.

(*Keywords: Ion-interaction model; Electrolytes; Apparent molal volumes; Apparent molal expansibility*)

*Eine alternative Methode zur Abschätzung der Pitzer-Koeffizienten der effektiven molalen Expansibilitäten von wässrigen Elektrolyten*

Es wird ein alternativer Weg zur Abschätzung der *Pitzer*-Koeffizienten nach dem ionischen Wechselwirkungsmodell für die effektiven molalen Expansibilitäten aus der Temperaturabhängigkeit der *Pitzer*-Koeffizienten für das effektive molale Volumen beschrieben. Die berechneten Koeffizienten sind in ausgezeichneter Übereinstimmung mit Werten, die nach einer Methode der kleinsten Fehlerquadrate aus experimentellen Daten erhalten wurden.

### **Introduction**

Very recently, we [1, 2] demonstrated the utility of the ion-interaction theory of *Pitzer* [3] in describing the volume properties of aqueous single and mixed electrolyte solutions. We fitted the volumetric data to appropriate individual *Pitzer* equations for apparent molal volume,  $\Phi_V$ , compressibility  $\Phi_K$  and expansibility  $\Phi_E$  of numerous aqueous single electrolytes and presented temperature and pressure dependences of the *Pitzer* ion-interaction coefficients. These coefficients for each of the properties of an electrolyte were evaluated employing a nonlinear least

squares technique. This note describes an alternative method to estimate  $\Phi_E$  using the temperature dependent equations for the *Pitzer* coefficients of  $\Phi_V$ .

### Results and Discussion

The *Pitzer* equations relating  $\Phi_V$  and molality,  $m$  ( $\text{mol kg}^{-1}$ ) for a single electrolyte type  $M\nu_M + X\nu_X^-$  is

$$\Phi_V = \Phi_V^0 + \nu |Z_M Z_X| (A_V/2b) \ln(1 + bI^{1/2}) + 2\nu_M \nu_X RT [m B_{MX}^V + m^2 (\nu_M \nu_X)^{1/2} C_{MX}^V] \quad (1)$$

with

$$B_{MX}^V = (\partial \beta^{(0)}/\partial P)_T + (\partial \beta^{(1)}/\partial P)_T (2/\alpha^2 I) [1 - (1 + I^{1/2}) \exp(-\alpha I^{1/2})] \quad (2)$$

and

$$C_{MX}^V = (\partial C^\Phi/\partial P)_T/2 \quad (3)$$

$\Phi_V^0$  is the value of  $\Phi_V$  at infinite dilution;  $\nu$ , stoichiometry of an electrolyte is  $\nu_M + \nu_X$ ;  $A_V$ , *Pitzer-Debye-Hückel* slope;  $\alpha$  and  $b$  are 2 and 1.2 respectively.  $I$  is the ionic strength on molal basis and  $R$  and  $T$  have their usual significance.  $(\partial \beta^{(0)}/\partial P)_T$ ,  $(\partial \beta^{(1)}/\partial P)_T$  and  $(\partial C^\Phi/\partial P)_T$  are the ion-interaction coefficients; we observed  $(\partial \beta^{(1)}/\partial P)_T$  to be negligible in all the electrolytes.

The apparent molal expansibility  $\Phi_E$  is given by

$$\Phi_E = (\partial \Phi_V/\partial T)_P \quad (4)$$

The pertinent ion-interaction coefficients for  $\Phi_E$  are  $(\partial^2 \beta^{(0)}/\partial P \partial T)$  and  $(\partial^2 C^\Phi/\partial P \partial T)$  with  $(\partial^2 \beta^{(1)}/\partial P \partial T)$  being zero. Earlier these coefficients were determined with a least squares routine [1, 2].

The temperature dependence of  $(\partial \beta^{(0)}/\partial P)_T$  and  $(\partial C^\Phi/\partial P)_T$  for  $\Phi_V$  was noted to fit with the expression

$$Y = a + bT + cT^2 \quad (5)$$

Thus the differentiation of Eq. (5) with respect to  $T$  should yield the ion-interaction coefficients for  $\Phi_E$ . So one has

$$(\partial Y/\partial T) = b + 2CT \quad (6)$$

Eq. (6) can be used for calculating  $(\partial^2 \beta^{(0)}/\partial P \partial T)$  and  $(\partial^2 C^\Phi/\partial P \partial T)$

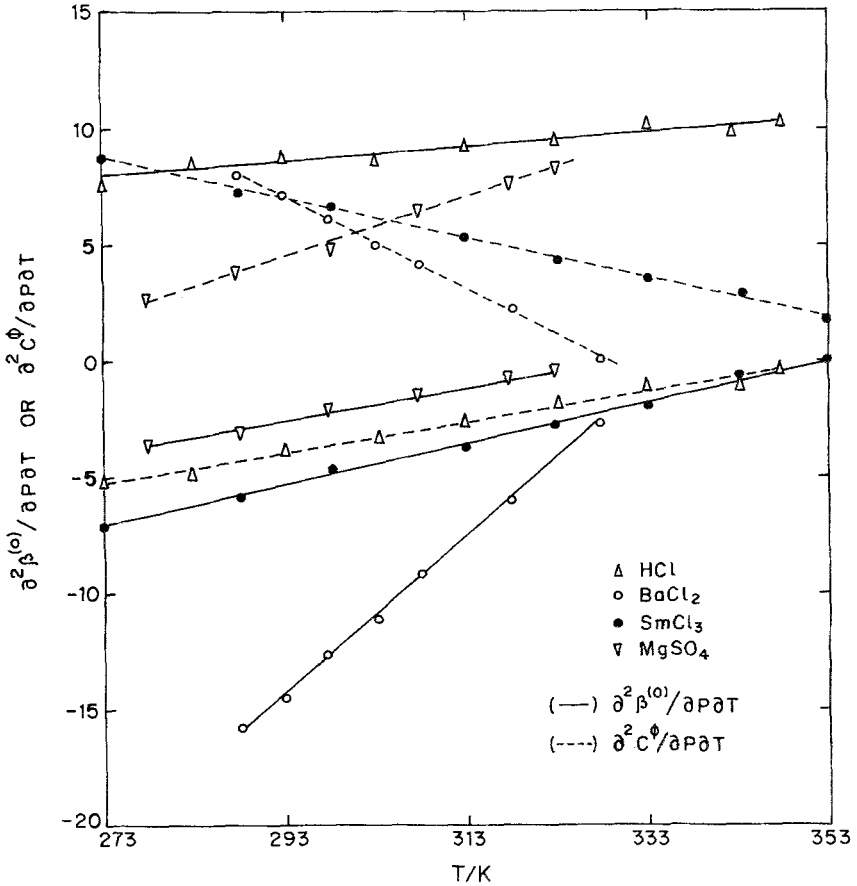


Fig. 1.  $\frac{\partial^2 \beta^{(0)}}{\partial P \partial T}$  and  $\frac{\partial^2 C^\Phi}{\partial P \partial T}$  as a function of  $T$  for single electrolyte solutions. Values of  $\frac{\partial^2 C^\Phi}{\partial P \partial T}$  to be multiplied by  $10^{-8}$  for HCl and SmCl<sub>3</sub>; both coefficients to be multiplied by  $10^{-6}$  for MgSO<sub>4</sub>. Remaining values to be multiplied by  $10^{-7}$ .

from the values of  $b$  and  $c$  parameters in the Table 2 of our earlier work [1].

The values of  $(\frac{\partial^2 \beta^{(0)}}{\partial P \partial T})$  and  $(\frac{\partial^2 C^\Phi}{\partial P \partial T})$  thus obtained agree closely to those obtained from the experimental data for reported 23 electrolytes of different charge type using a least squares routine. In Fig. 1 the calculated values of these coefficients are plotted for selected electrolyte solutions HCl, BaCl<sub>2</sub>, MgSO<sub>4</sub>, SmCl<sub>3</sub> along with those obtained through a direct  $\Phi_E$  fit, showing excellent agreement. The average standard deviation of fit for 23 electrolyte solutions is 0.9%.

These coefficients can be easily substituted in the appropriate  $\Phi_E$  equation in order to estimate  $\Phi_E$  at various concentrations and temperatures without a least squares program.

### References

- [1] *Kumar A* (1987) *J Chem Eng Data* 32: 106
- [2] *Kumar A* (1987) *J Chem Eng Data* 32: 109
- [3] *Pitzer KS* (1973) *J Phys Chem* 77: 268